Smoothness of Lyapunov Exponents as Functions of Perturbation Parameters

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Abstract

Using powerful spatial averaging techniques we develop an expression for the derivative of the Lyapunov exponents with respect to the coupling parameter for a chaotic Hamiltonian system. The expression predicts that the Lyapunov exponents will be smooth functions of the parameter under some fairly strict, but not unreasonably rare, conditions. We will present some supporting numerical work and discuss the limits of validity for this approach.

The Derivative

There are four terms in the derivative of Eq. 7. Assuming no regular regions exist[5], the variables of integration are constrained by the argument of the step function, $D = E + \hat{p}_f^2/2$. The derivative can be compactly expanded as

$$\frac{d\lambda}{d\alpha} = \frac{\lambda}{2D} \sum_{\delta} \left\{ \frac{d\delta}{d\alpha} \right\}$$

The first three terms are expected to, and do, converge rapidly using a Monte Carlo integration. The fourth term, involving $d\beta/d\alpha$, is likely to involve singularities in the vicinity of $\alpha = 0$. There are no visible regular regions when $\alpha = 0.6$. At left is a time computation of the Lyapunov exponent. Much of the variation in $\lambda$ can be attributed to either numerical error or cantori[1].

Concluding Thoughts

Eight order Runge-Kutta integration to find $s$ and $\partial s/\partial \alpha$ followed by Monte Carlo integration on the four terms in the derivative of Eq. 7 show that the first three terms converge sufficiently quickly to have computational use. The fourth term, however, has not responded well to Monte Carlo techniques. Nonetheless, an analysis of the behavior of the $s$ field under small perturbations yield qualitative evidence that the Lyapunov exponent for at least the coupled quartic system may be differentiable for certain regions of $\alpha$.

References