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Game Analysis of Mu Torere & Related Ethnographic Games Work

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MAA Session on Ethnomathematics: A Tribute to Marcia Ascher

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Marcia Ascher studied the game of Mu Torere in her 1987 paper “Mu Torere: An Analysis of a Maori Game”, and we will continue this study here.

This game is usually represented as being played on a board like this, with a center point called the “putahi”, 8 “star” points surrounding that, and 4 pieces for each of 2 players, set on those star points. A Mu Torere board that was purchased from the Canterbury Museum in Christchurch, New Zealand shows these points as the 8 legs of an octopus, and this shows the traditional starting position with 4 pieces of each color surrounded by the legs. While this is an attractive board, there seems to be no reason to believe this octopus was a traditional view of the game board.

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In playing the game, players alternate moves which can be: (a) from a star point to an empty adjacent star point; (b) from the center to an empty star point; or (c) from a star point to an empty center if the piece being moved is adjacent to a star point with an opponent’s piece. SHOW Thus this would be a legal move, but this (which would immediately win the game) is not. Some sources say that this constraint on (c) applies only for the first two moves, and it remains unclear whether both variations were played historically. A player wins the game if, they leave their opponent with no legal move to make, i.e. if they are “blocked” from access to the unique open point on the board.

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To compare this to other similar games, we convert it to an isomorphic graph game, where we replace the traditional board with a stellated octagon. The moves on this board can be described simply: You move a piece along an incident edge to an unoccupied vertex – with the constraint above about moving to the vertex of degree 8.

This now fits into a reasonably well-populated category of “Blockade games”, in which you move pieces along the edges of a graph trying to block your opponent so they have no moves.

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On the right is a game called “Pong Hau K’i”, from northern China, showing the starting position for the game, with 2 players and one open vertex. On the left is a picture from a Korean schoolbook showing two historical Korean games of the “Gonu” family, both of which are blockade games. Umul Gonu is equivalent to Pong Hau K’i except for the starting position. Because of the starting position, it uses the same constraint as Mu Torere on the first move (but only the first move). SHOW: Thus this piece can’t move to the center on the first move.

Equivalent games, with a different way to start, are played in Thailand (Sua tok tong) and in northern India (Punjab: Do-guti). The game Pat Gonu always has several open spots, as do several blockade games from other countries, but we will not be looking at them today. There is one more blockade game with a single open vertex that we will show shortly.

In her paper, Ascher argued that the game of Mu Torere played on a size other than an octagon would not be an interesting game, i.e. either impossible to win, or too easy to win. However, she assumed that the game would use a complete square, hexagon, etc. The game of “Pong Hau K’i” is essentially identical to Mu Torere except that it is played on a square with one side missing.
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Similarly, when this game is played on a full hexagon, it’s not interesting, but when played on a hexagon with one missing edge, it is. In fact, exactly this game was invented by the General Mills advertising folks as a children’s game for the back of a “Berry Berry Kix” box, a game which they called “Berry Patch Scramble”. The starting position here does not require the extra constraint on playing to the center.

Phil Straffin talked to the game designers about this game, and they had no idea that there were other games of this style, so it appears to be a case of independent invention.

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In her 1987 paper, Ascher constructed this “game chart” for Mu Torere showing the possible states of the game, and the moves which can be made from one state to another. We can analyze the game using this chart and establish such facts as that, with best play, it is a draw. She makes this claim in her paper, but doesn’t actually justify that claim, possibly because of the complexity of her chart. Ascher’s game chart is complex enough to make it difficult to discover certain features of this game. Phil Straffin and I used this game, and this chart, in teaching a course in Ethnomathematics at Beloit College – a class which we alternated teaching for more than 20 years. After a few years of this, Phil discovered an easier chart to work with.
The Ascher chart encodes both the current positions of the pieces and which player will move next into one of the nodes in her chart, and then uses a directed graph to represent the moves.

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Phil’s change was to encode only the position of the pieces into the vertices, and then label the graph edges as “A” or “B” depending on which player could move from one game position to another. This reduces the number of vertices by about half, and converts directed edges into undirected edges. The complication of labeled edges turns out to be a minor inconvenience in the resultant analysis.

I think this graph is substantially easier to look at, and to analyze. In particular, Straffin noticed the significance of the dashed line shown. The game starts to the left of the dashed line, and all of the winning positions (shown in bold) are on the left side. So long as the play remains on the left, the game is competitive, with neither player having an advantage, and either player able to win if their opponent errs. If you move to the right of that line, though, the game becomes a draw with reasonable players. Specifically, if you’re on the right of that line, and make a move to the left, you will immediately lose to any player with reasonable skill; but you are never forced to move to the left — there is always an alternative move that keeps you on the right.

Before we look at this chart in more detail though, let’s look at those for the simpler Blockade games.

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The position chart for Pong Hau K’i is shown here. With my Ethnomath students, I have them do an analysis of this game by imagining what would happen if the players had perfect 1-ply lookahead. Then A would never make this move, because a different move would win. If we remove these “impossible” moves, we get this reduced position chart. Now these moves are unreachable, so we delete them as well. And this Position Chart is pretty easy to analyze: If you have 2-ply lookahead, A would certainly never make this move, so B will never win (and vice versa). Thus for players with perfect 2-ply lookahead, the chart reduces to this. Since there is no longer a path from the start position to either of the winning positions, the game is a draw, at least for players at this skill level. This also shows the idea of a position that sets a “trap” for the other player: If A regularly tries to play to this position, it’s a trap for B – B might make the wrong move and let us win.

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Next, we’ll do a similar analysis for Berry Patch Scramble. This Position Chart is more complex than the one for Mu Torere: Since there is less symmetry in the game board, there are more positions in the chart. Again, Phil built the Position Chart for this game, and found the line here separating the “draw territory” from the “win
territory”, with winning positions shown in dark. For a simple analysis of this game, we start by looking at the 1-ply lookahead chart. This already shows us the importance of Straffin’s dividing line: If you ever cross that line, the game will quickly end, but you always have the alternative to stay in “draw territory”. The 2-ply lookahead chart makes a final analysis even more clear-cut. At this point, we can see that perfect play will require 4-ply lookahead to stay on the draw side of the chart. It also shows us the most important “trap” positions: You try to play to these positions, and give your opponent chances to make a mistake.

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Finally, let’s return to Straffin’s Position Chart for Mu Torere. Here, the starting position for the game is on the same side of the dividing line as the winning positions. For simplicity, I assume we’re playing the version where the extra condition on moving to the center remains in effect for the entire game; the other case is similar. When we go to the 1-ply Lookahead Chart, we get a phenomenon of edges that aren’t completely erased: They are still viable to play from one direction, but not from the other. Instead of trying to show directional edges, I’ve simply shown them as edges that are only connected to vertices on one side. Show: So it’s reasonable for A to play from here to here, but not the reverse, since that would lead to a loss. The 1-ply lookahead chart looks simpler, but still complex. I’ve had to move the starting position, which has been erased, but the first three moves are forced, so that’s easy to assume. With the 2-ply lookahead chart, each player has lost all but one of their winning positions – the others can never be reached by players at this skill level. 3-ply lookahead doesn’t simplify things much more, and there are still a few links going back and forth between the right and left side of this chart, so at this skill level you might still play from the right to the left. The 4- and 5-ply lookahead charts are identical, and the right side has become an absorbing state: If you move to that side, you’ll never come back. Only with the 6-ply lookahead chart do we lose all access to the winning positions. Here we can get into infinite draws on either the left side or on the right side. So with perfect play, e.g. with 6-ply lookahead, Mu Torere is a draw, and with 4-ply lookahead, the right side becomes a guaranteed draw. The 4-ply lookahead chart also shows us the best “trap” positions to play, hoping that our opponent occasionally fails to see 6-ply ahead.

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Marcia Ascher began the analysis of these Blockade Games using Game Charts. Phil Straffin found an easier way to analyze this type of game through his Position Charts. The analysis of Pong Hau K’i was straight-forward, and our students do this in our classes, but this improved technique was probably what allowed him to complete the analysis of Berry Patch Scramble. It almost certainly is what allowed him to discover the “Straffin dividing lines” in these three games. I hope my use of the Lookahead Position Charts makes understanding some of the complexities of these games easier. Using these with my students seems to imply that this is the case.

Slide 13: References:
Marcia Ascher, “Mu Torere: An Analysis of a Maori Game”,
P.D. Straffin, "Corrected Figure for Position Graphs for Pong Hau K’i and Mu Torere," Mathematics Magazine 69 (1996) 65.